EFFECT OF PHASE VELOCITY DIFFERENCE ON THE TURBULENT STRUCTURE OF A JET CARRYING HEAVY IMPURITIES

T. A. Girshovich and V. A. Leonov

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A model is proposed to calculate the effect of a difference in the mean velocities of particles and a gas on the turbulent structure of a jet carrying heavy impurities. The model considers the effect of all particles in a mole of the gas on the velocity of the mole at the end of its "life."

The effect of flow nonequilibrium — by which we mean the existence of a difference in the mean velocities of a gas and the particles in the gas flow — was examined in [1]. According to the model proposed in this work, the initial velocity of the particles in a mole is assumed to be equal to the difference in the mean velocities of the phases. It is also assumed that a given mole retains the particles inhabiting it. The longitudinal velocity components of the mole and particles are affected in one direction or the other by the phase velocity difference, depending on the sign of the velocity of the mole. The phase velocity difference exerts no effect on the transverse eddy velocities of the particles and gas. The model proposed in [1] has a serious flaw — it does not account for the fact that the particles may leave the mole due to the phase velocity difference, and that new particles may enter the mole. Thus, this model is applicable only in the case when the phase velocity difference is small and is less in absolute value than the velocity of the mole.

An attempt is made in [2] to determine the effect of a difference in phase velocity on the transverse component of the mole velocity. The variability of the mass of the particles is considered here. It is assumed that all of the particles in a given mole interact with the mole. However, it is also assumed that the momentum of the mole is spent entirely on imparting momentum to the particles and on the change in mass. Obviously, the particles given momentum here are present in the mole.

Examined below is a model describing the effect of flow nonequilibrium on the turbulent structure of a jet. As in [2], we consider the effect of all particles in a mole on the velocity of the mole at the end of its life.

Since, as noted above, the momentum of the mole is spent entirely on changing the velocity of particles within the mole, the equation of momentum of the mixture of solid particles and gas has the form

$$m_g \frac{dv_g'}{dt} = - m_p \frac{dv_p'}{dt}.$$

(1)

With a nonequilibrium flow, the mass of the particles in the mole is dependent on time.

We will assume (as in [2]) that the mole is a cube with the side a. Then the mass of the particles which have passed through the mole up to the time t, reckoned from the beginning of its life, can be determined from the formula

$$m_p = \rho a^3 \varkappa + \int_0^t \rho a^2 \varkappa |u_g - u_p| dt.$$

(2)

If we assume that u_g, u_p, and $\varkappa$ are constant along the path of motion, then (2) takes the form

$$m_p = \rho a^3 \varkappa + \rho a^2 \varkappa |u_g - u_p| t.$$

(3)

The quantity $u_g - u_p$ is taken as a modulus because the mass which passes through the mole does not change in relation to the direction of its passage -- left to right or right to left. The mass of the gas mole is equal to $m_g = \rho a^3$.

Integration of Eq. (1) — which should be done with allowance for the fact that m_p is dependent on time — gives us

$$v'_{gi} - v'_{g0i} = - \varkappa (v'_p - v'_{p0}) - \varkappa \frac{|u_g - u_p|}{a} \int_0^t t \frac{dv'_p}{dt} dt.$$

Integrating by parts on the right side of this equation, we obtain the following after several simple transformations:

$$v'_{gi} - v'_{g0i} = - \varkappa (v'_p - v'_{p0}) - \varkappa \frac{|u_g - u_p|}{a} t v'_p + \varkappa \frac{|u_g - u_p|}{a} \int_0^t v'_p dt. \tag{4}$$

Here, v'_{gi} and v'_{g0i}, v'_p and v'_{p0} are the velocities of the mole and particles at the end and beginning of the integration interval, respectively.

The time a particle spends in the mole in the case of its transverse movement is equal to

$$\tau = a/|u_g - u_p|. \tag{5}$$

The equation of motion of the particle in the Stokes approximation has the form

$$\frac{dv'_p}{dt} = N(v'_g - v'_p), \quad N = 18\mu_g/D_p^2\rho_s. \tag{6}$$

We will designate

$$v'_\sim = v'_g - v'_p. \tag{7}$$

Solving (4) and (6) together and using (7), we obtain an equation to determine $v'_\sim$:

$$\frac{1}{v'_\sim} \frac{dv'_\sim}{dt} = - N \left[1 + \varkappa + \varkappa \frac{|u_g - u_p|}{a} t \right]. \tag{8}$$

Integration of this equation with the initial condition $v'_\sim = v'_{\sim 0i}$ and $t = \tau = 0$ gives the following from $t = 0$ to $t = \tau$:

$$v'_\sim = v'_{\sim i} \exp \left\{ - N\tau \left[1 + \varkappa + \frac{\varkappa |u_g - u_p|}{2a} \tau \right] \right\} = v'_{\sim i} \exp [- F(\tau)]. \tag{9}$$

Each group of particles which enters the mole is accelerated within the mole from an initial velocity v'_{p0} to a certain higher velocity in accordance with Eq. (9), with a certain amount of the momentum of the gas being used in the process. For simplicity, the continuous exchange of momentum between the particles and the gas in the case of a continuously changing particle composition can be represented as a piecewise-continuous process. Here, the entire life of the mole t_m is broken down into segments τ. The mass of the particles which pass through the mole in each of these time segments changes in accordance with Eq. (3) from $m_{p0} = \rho a^3 \varkappa$ to $m_p = \rho a^3 \varkappa + \rho a^2 \varkappa |u_g - u_p| \tau$. The velocity of the particles accounting for this mass changes from v'_{p0} to the velocity that they reach after the time τ. Accordingly, the velocity of the gas changes continuously from the velocity v'_{g0} at $t = 0$ to the velocity which the gas mole reaches after the time τ as a result of its interaction with the particles. Then the quantity $v'_\sim$ is piecewise-continuous during the intervals τ which make up the lifetime of the mole. The quantity $v'_\sim$ changes discontinuously at the beginning of each such interval, while the quantity v'_g changes continuously. The value of $v'_{\sim 0i}$ depends on time and changes n times during the life of the mole (n is the integral part of the ratio t_m/τ).

At the end of the n-th interval, we have

$$v'_{\sim n} = \sum_{i=1}^{n-1} (v'_{pi} - v'_{p0}) \exp [-(n-i)F] + v'_{\sim 0} \exp (-nF). \tag{10}$$

At the end of the mole's life, $v'_\sim$ is determined from the formula

$$v'_{\sim k} = v'_{\sim 0} \exp\left[-nF - F(\Delta t)\right] + (v'_p - v'_{p0})\exp\left[-F(\Delta t)\right] + \sum_{i=1}^{n-1}(v'_{pi} - v'_{p0})\exp\left[-(n-i)F - F(\Delta t)\right]. \quad (11)$$

Here

$$\Delta t = t_m - n\tau. \quad (12)$$

It is apparent from (11) that determination of $v'_{\sim k}$ requires knowledge of the velocity of the particles at the end of each period τ. This quantity can be found by means of Eq. (11). We find from this equation that

$$v'_g - v'_{g0} = -\varkappa(v'_p - v'_{p0}) - \frac{\varkappa t}{\tau}v'_p + \frac{\varkappa}{\tau}\int_0^t v'_p dt. \quad (13)$$

At the end of the first interval, the velocity of the gas mole is equal to

$$v'_{g1} = v'_{p1} + v'_{\sim 0}\exp(-F).$$

We will insert v'_g from this expression into (13). After a few transformations, we obtain

$$v'_p = \frac{1}{1 + \varkappa + \varkappa\dfrac{t}{\tau}}\left[v'_{g0} + \varkappa v'_{p0} + \varkappa\frac{y_p}{\tau} - v'_{\sim 0}\exp(-F)\right], \quad (14)$$

where y_p is the movement of the particles during the time t

$$y_p = \int_0^t v'_p dt. \quad (15)$$

To calculate y_p, we need to integrate the equation

$$\frac{dy_p}{dt} = \frac{1}{1 + \varkappa + \varkappa\dfrac{t}{\tau}}\left(v'_{g0} + \varkappa v'_{p0} - v'_{\sim 0}\exp\left[-Nt\left(1 + \varkappa + \frac{\varkappa t}{2\tau}\right)\right] + \frac{\varkappa y_p}{\tau}\right). \quad (16)$$

Without introducing a large error, we can simplify Eq. (11) in a first approximation by setting

$$v'_{p1} = v'_{p2} = \ldots = v'_{pn}. \quad (17)$$

Then (11) takes the form

$$v'_{\sim k} = v'_{\sim 0}\exp\left[-nF - F(\Delta t)\right] + (v'_p - v'_{p0})\sum_{i=1}^{n}\left[-(n-i)F - F(\Delta t)\right]. \quad (18)$$

We find the quantity v'_{gk} using Eq. (18):

$$v'_{gk} = v'_{pk} + v'_{\sim 0}\exp\left[-nF - F(\Delta t)\right] + (v'_p - v'_{p0})\exp\left[-F(\Delta t)\right]\sum_{i=1}^{n}\exp\left[-(n-i)F\right], \quad (19)$$

where v'_{pk} is determined from Eq. (14), in which, in place of t, we have substituted τ.

If the flow is equilibrium, then $u_p \to u_g$, $\tau = a/|u_g - u_p| \to \infty$, $n = (t_m - \Delta t)/\tau \to 0$, $F(\Delta t) = -N\Delta t(1 + \varkappa + \varkappa\Delta t/2\tau) \to -N\Delta t(1 + \varkappa)$ and $v'_{\sim k}$, v'_{pk} and v'_{gk} take the form

$$v'_{pk} = \frac{1}{1 + \varkappa}(v'_{g0} + \varkappa v'_{p0} - v'_{\sim k}), \qquad v'_{gk} = \frac{1}{1 + \varkappa}(v'_{g0} + \varkappa v'_{p0} - v'_{\sim k}) + v'_{\sim k}$$

or

$$v'_{gk} = \frac{1}{1 + \varkappa}(v'_{g0} + \varkappa v'_{p0} + \varkappa v'_{\sim k}),$$

from which we obtain the formula for the velocity of the gas mole at the end of its life that was derived in [1] (see (1.2)) or the formula obtained in [3] for particles of moderate size ($v'_{po} = 0$):

$$v'_g = v'_{g0} \frac{1 + \varkappa \dfrac{v'_\sim}{v'_{g0}}}{1 + \varkappa}. \tag{20}$$

As noted above, the transverse velocity component of the particles at the beginning of the life of the mole v'_{po} depends only on the relative size of the particles. This is the velocity which the particles have at the end of the formation of the mole. Obviously, the longitudinal component of the eddy velocity of the particles at the beginning of the life of the mole is equal to the sum of the velocity of the particles at the end of mole formation and the difference between the mean longitudinal velocities of the particles and gas:

$$u'_{p0} = u'_{p00} + (u_p - u_g). \tag{21}$$

The positive and negative transverse components of eddy velocity are the same in absolute magnitude. Since the sign of the initial longitudinal velocity of the particles in the mole is independent of the sign of the eddy velocity of the gas, then the longitudinal components of the eddy velocities of the gas and particles at the end of the life of the mole should depend on the direction of motion of the mole. The rms values of gas and particle velocity in this case can be determined from the formulas [1]

$$u'_g = \frac{u'^{+}_g - u'^{-}_g}{2}, \quad u'_p = \frac{u'^{+}_p - u'^{-}_p}{2}. \tag{22}$$

The quantities $u'^{\pm}_g$ and $u'^{\pm}_p$ are, respectively determined from (19) and (14), in which $u'^{\pm}_{\sim o}$ and $u'^{\pm}_{go}$ (the same quantities, with their signs) are taken as the initial values.

The time of interaction of each particle group with the mole in pulsative longitudinal motion is different from the time of interaction of the particles with the mole in transverse pulsative motion. This time increases if the signs of u'_{po} and u'_{go} coincide and decreases if they differ. The time of interaction of the particles and mole in longitudinal pulsative motion can be determined from the formula

$$\tau_{\pm} = \frac{a}{|u'_{p\,av} + u_p - u_g - u'^{\pm}_{g\,av}|}$$

or, without introducing a large error, it can be set equal to

$$\tau_{\pm} = \frac{a}{|u'_{p00} + u_p - u_g - u'^{\pm}_{g0}|}. \tag{23}$$

The pulsative (eddy) components of the particle and gas velocities at the end of the mole lifetime at a fixed point of the cross section of the jet can be calculated in the following sequence:

1) the lifetime of the mole is calculated

$$t_m = \frac{l_u}{|v'_{g0}|} = \frac{1}{\left| \dfrac{\partial u}{\partial y} \right|}, \tag{24}$$

where

$$v'_{g0} = l_u \frac{\partial u}{\partial y},$$

and the time of interaction of the particles with the mole is calculated from Eq. (5) (for transverse pulsative motion) or Eq. (22) (for longitudinal pulsative motion); the velocity profile can be assigned using Schlichting's formula

$$\frac{u}{u_m} = (1 - \eta^{3/2})^2, \quad \eta = y/\delta; \tag{25}$$

2) the quantity n is determined as the integral part of t_m/τ, and the quantity Δt is determined from the formula

$$\Delta t = t_m - n\tau;\qquad(26)$$

3) the quantities $\eta_\varkappa$ and $\varkappa$ are determined from the formulas*

$$\eta_\varkappa = \frac{y}{\delta_u}\,\frac{\delta_u}{\delta_\varkappa},\quad \varkappa = \varkappa_m(1 - \eta_\varkappa^{3/2})^2;\qquad(27)$$

4) the quantities a and $F(t)$ are determined from the formulas

$$a = l_u = \delta\beta,\text{ where } \beta = l_u/\delta \simeq 0.09;\qquad(28)$$

$$F(t) = N\tau\left[1 + \varkappa + \frac{\varkappa\,|u_g - u_p|}{2a}\,\tau\right];$$

5) the movement of the particles during the time τ is determined by integrating the differential equation

$$\frac{dy}{dx} = \frac{\tau}{1 + \varkappa + \varkappa x}\left\{v'_{g0} + \varkappa v'_{p0} - v'_{\sim 0}\exp\left[-Nx\left(1 + \varkappa + \frac{\varkappa x}{2}\right)\tau\right] + \frac{\varkappa y}{\tau}\right\}$$

with the initial condition y = 0 at x = 0; here, the value of x changes from 0 to 1.

6) the quantities $v'_{\sim k}$ and v'_{gk} are determined from Eqs. (18) and (19) (the quantity Δt can be set equal to zero as a first approximation).

Figure 1 shows the relative eddy velocities of the gas over the cross section of the jet. The calculations were performed for a flow with mean gas and particle velocities of 30 and 40 m/sec, an impurity concentration of 2, and particles 40 μm in size. The quantity V° represents the pulsative (eddy) components of the gas velocity referred to the transverse eddy velocity of the gas mole at the beginning of its life; η is the distance from the jet axis over a radius of the cross section, referred to the radius. Curve 1 corresponds to the distribution of the longitudinal eddy velocity of the gas for the case of flow of a gas and impurity particles with the same velocity (equilibrium flow). Curves 2 and 3 correspond to the distribution of the transverse and longitudinal pulsative components of gas velocity for nonequilibrium flow. It can be seen from the graph that flow nonequilibrium leads to a reduction in the relative longitudinal and transverse pulsative components of the gas velocity. Here, the longitudinal component decreases more rapidly than the transverse component.

Figure 2 shows the results of calculation of the shear stress distribution in the gas over the jet cross section at the end of the mole lifetime. The values are referred to the shear stresses in the jet in the absence of impurities τ°. The calculations were performed for u_{gm} = 30 m/sec, u_{pm} = 40 m/sec, $\varkappa$ = 2, and different particle diameters. Curves 1, 2, and 3 correspond to sizes of 50, 40, and 30 μm. The dashed curve corresponds to the shear stress distribution in the gas for the equilibrium flow of a two-phase jet with 50-μm particles.

Figure 3 shows the effect of impurity concentration on the relative shear stress distribution in the gas at the end of the life of a mole for 40-μm particles and average phase velocities u_{gm} = 30 m/sec and u_{pm} = 40 m/sec. Curves 1 and 2 correspond to impurity concentrations of 2 and 4. The dashed curves correspond to the shear stresses in the gas in the case of equilibrium flow. It is apparent from the graphs that a decrease in the size and an increase in the concentration of the particles lead to an increase in the effect of the particles on the turbulent structure of the jet. The presence of nonequilibrium in this case leads to a reduction in shear stresses.

Figure 4 shows the effect of the degree of nonequilibrium on the shear stresses in the gas at the end of the mole lifetime. The calculations were performed for a mean gas velocity of 30 m/sec, particle size of 40 μm, and impurity concentration of 2. Curves 1 and 2 correspond to mean particle velocities of 40 and 60 m/sec. The dashed curve depicts the shear stress distribution for equilibrium flow. It is apparent from the graph that an increase in

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*It should be noted that the quantity $\delta_u/\delta_\varkappa$ can be assigned on the basis of considerations discussed in [3-5] regarding the diffusion of particles in a turbulent jet.
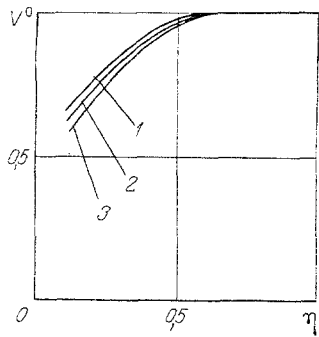
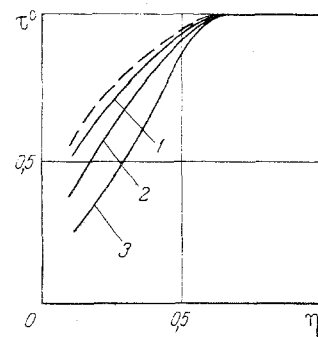Fig. 1   Fig. 2

Fig. 1. Change of relative pulsative components of longitudional and transverse velocities of gas across the jet: 1) equilibrium flow; 2,3) transverse and longitudinal components of eddy velocity with nonequilibrium flow (u_{gm} = 30 m/sec, u_{pm} = 40 m/sec).

Fig. 2. Effect of particle diameter and nonequilibrium of flow on distribution of relative shear stresses in the gas across the jet: 1) D_p = 50; 2) 40; 3) 30 μm; dashed line) equilibrium flow with D_p = 50 μm.
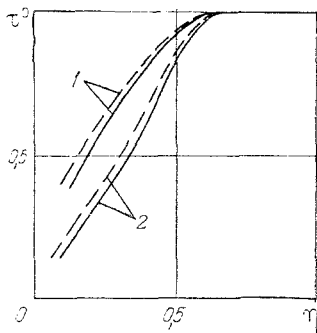

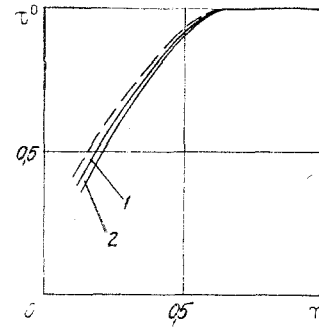Fig. 3   Fig. 4

Fig. 3. Effect of impurity concentration and flow nonequilibrium on the distribution of relative shear stresses in the gas across the jet: 1) $\varkappa$ = 2; 2) 4; dashed curves) equilibrium flow.

Fig. 4. Effect of phase velocity difference on distribution of relative shear stresses across the jet: 1) u_{gm} = 30 m/sec, u_{pm} = 40 m/sec; 2) u_{gm} = 30 m/sec, u_{pm} = 60 m/sec; dashed curve) equilibrium flow.

the degree of flow nonequilibrium reinforces the effect of impurities on the turbulent structure of the jet.

NOTATION

a, length of side of cube modeling a turbulent mole; D_p, particle diameter; l_u, displacement path; G_n, G_g, flow rates of impurity and gas; m_g, m_p, masses of gas mole and particles; t, running time; t_m, lifetime of mole; u_p, u_g, longitudinal components of mean velocities of particles and gas; u'_{po}, u'_{go}, pulsative components of longitudinal velocities of particles and gas at beginning of mole lifetime; u'_p, u'_g, pulsative components of longitudinal velocity of particles and gas; v'_p, v'_g, pulsative components of transverse velocity of particles and gas; v'_{po}, v'_{go}, pulsative components of transverse velocity of particles and gas at beginging of mole lifetime; $v'_{\cdot}$, relative velocity of gas; x, y, cartesian coordinates; x, directed along jet axis; y, directed perpendicular to axis; y_p, displacement of particles; δ_u, $\delta_\varkappa$, ordinate of jet boundary with respect to velocity and concentration; $\varkappa = G_n/G_g$, concentration of impurities; μ_g, viscosity of gas; ρ_g, ρ_s, ρ, density of gas, particles, and gas-particle mixture, respectively; τ, residence time of particle in mole.

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METHOD OF CALCULATING LIMITS OF STABILIZATION OF A FLAME OF
INHOMOGENEOUS MIXTURES TRAVELING OVER A POORLY STREAMLINED BODY

V. L. Apollonov, V. N. Gruzdev,   UDC 536.46:662.612.31:662.942
and A. V. Talantov

A method has been developed for calculating the range of stabilization of a flame of inhomogeneous mixtures with allowance for characteristics of the atomization, vaporization, and distribution of the fuel in the flow.

The empirically established fact that the range of stable combustion of inhomogeneous mixtures can be generalized using the excess air coefficient for the vapor-phase circulation zone [1] can serve as a starting point for deriving an equation to calculate the total excess-air coefficient α at the moment of flameout. The range of stabilization of the flame in the channel is actually evaluated from the total quantity α.

The total amount of fuel in the vapor phase which enters the circulation zone per unit of time is made up of the fuel vaporized in the flow in the section from the nozzle to the stabilizer and the fuel which diffuses into the circulation zone as a result of vaporization on the hot stabilizer:

$$g_{c.z} = g_n + g_{st}. \tag{1}$$

Here

$$g_n = c\varphi g_f \frac{G_{c.z}}{G}, \tag{2}$$

$$g_{st} = g_f (1 - \varphi) m\theta_{tot}. \tag{3}$$

Writing (1)-(3) in terms of the equivalent ratios, we obtain

$$\frac{\alpha}{\alpha_{c.z}} = c\varphi + (1 - \varphi) m\theta_{tot} \frac{G}{G_{c.z}}. \tag{4}$$

To determine the form of the exchange function, we need to know the exact amount of fuel deposited on the stabilizer and the amount then transferred from the surface of the stabilizer to the circulation zone. For this purpose, fuel was delivered directly onto the rear surface of the stabilizer, rather than added to the flow. This prevented the fuel from vaporizing in the flow before it reached the stabilizer. In this case then, the composition of the mixture in the circulation zone owes only to the vaporization of the liquid fuel on the